THE

MATHEMATICAL GAZETTE.

EDITED BY

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WITH THE CO-OPERATION OF

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REPORT OF THE BRITISH ASSOCIATION COMMITTEE ON THE TEACHING OF MATHEMATICS.*

In submitting their present report, the Committee desire to point out that this is not the first occasion on which the British Association has attempted to deal with the teaching of elementary mathematics. About thirty years ago, a similar body was appointed to consider a part of the subject, viz., "the possibility of improving the methods of instruction in elementary geometry"; and two reports were presented, one at the Bradford meeting in 1873 (see the Report volume for that year, p. 459), the other at the Glasgow meeting in 1876 (see the Report volume for that year, p. 8).

The two reports advert to some of the difficulties that obstruct improvements in the teaching of geometry. One of these is alleged to be "the necessity of one fixed and definite standard for examination purposes"; apparently, it was assumed that this fixed and definite standard should not merely be required from all candidates in any one examination, but also be applied to all examinations throughout the country. In order to secure the uniformity thus postulated, the Committee, thinking that no text-book had been produced fit to succeed Euclid in the position of authority, and deeming it improbable that such a book could be produced by the joint action of selected individuals, suggested the publication of an authorised syllabus. In their second report, they discussed the merits of a particular syllabus—that of the Association for the Improvement of Geometrical Teaching; but, in spite of such commendation as was then expressed, the syllabus has not been generally adopted.

It is still true that (in the words of the former Committee) "in this country at present teaching is guided largely by the requirements of examinations." For some time to come, the practice of the country is not unlikely to allow examinations to retain at least a partial domination over

* Brought before the Educational Section of the Association at Belfast on September 12th. The Committee was appointed to report upon improvements that might be effected in the teaching of mathematics, in the first instance in the teaching of elementary mathematics, and upon such means as they think likely to effect such improvements. The members of the Committee were Prof. Chrystal, Mr. W. D. Eggar, Mr. H. W. Eve, Prof. Forsyth (Chairman), Prof. Gibson, Dr. Gladstone. Prof. Greenhill, Prof. R. A. Gregory, Prof. Henrici, Prof. Hudson, Dr. Larmor, Prof. A. Lodge, Principal O. Lodge, Prof. Love, Major MacMahon, Prof. Minchin, Prof. Perry (Secretary), Principal Rücker, Mr. Robert Russell, and Prof. S. P. Thompson. The report was drawn up by the Chairman.

teaching in schools. Accordingly, if the teaching is to be improved, it seems to be a preliminary requisite that examinations should be modified; and, where it is possible, these modifications in the examinations should leave greater freedom to the teacher, and give him more assistance than at present.

On the other hand, there is a tendency is this country whereby, in such matters as teaching and examination, the changes adopted are only gradually effected, and progress comes only by slow degrees. Accordingly, the general recommendations submitted in this report are such that they can be introduced easily and without any great alteration of the best present practice. It is the hope of the Committee that the recommendations, if adopted, will constitute merely the first stage in a gradual improvement both of teaching and of examinations. For the most part, only broad lines of change are suggested: this has been done in order to leave as much freedom as possible to teachers for the development of their methods in the light of their experience.

The Committee do not consider that a single method of teaching mathematics should be imposed uniformly upon all classes of students; for the only variations then possible would be limited by the individuality of the teacher. In their opinion, different methods may be adopted for various classes of students, according to the needs of the students; and corresponding types of examination should be used.

It is generally, if not universally, conceded that a proper training in mathematics is an important part of a liberal education. The value of the training depends upon the comprehension of the aims of the mathematical subjects chosen, upon the grasp of the fundamental notions involved, and upon the attention paid to the logical sequence of the arguments. On the other hand, it is freely claimed that, in the training of students for technical aims such as the profession of engineering, a knowledge of results and a facility in using them are more important than familiarity with the mathematical processes by which the results are established with rigid precision. This divergence of needs belongs, however, to a later stage in the training of students. In the earliest stages, when the elements of mathematics are being acquired, the processes adopted can be substantially the same for all students; and many of the following recommendations are directed towards the improvement of those processes.

The former Committee recommended (and the present Committee desire to emphasise the recommendation) that the teaching of demonstrative geometry should be preceded by the teaching of practical and experimental geometry, together with a considerable amount of accurate drawing and measurement. This practice should be adopted, whether Euclid be retained, or be replaced by some authorised text-book or syllabus, or if no authority for demonstrative geometry be retained.

Simple instruments and experimental methods should be employed exclusively in the earliest stages, until the learner has become familiarised with some of the notions of geometry and some of the properties of geometrical figures, plane and solid. Easy deductive reasoning should be introduced as soon as possible; and thereafter the two processes should be employed side by side, because practical geometry can be made an illuminating and interesting supplement to the reasoned results obtained in demonstrative geometry. It is desirable that the range of the practical course and the experimental methods adopted should be left in large measure to the judgment of the teacher; and two schedules of suggestions, intended for different classes of students, have been submitted to the Committee by Mr. Eggar and Professor Perry respectively, and are added as an appendix to this Report.

In the opinion of the Committee, it is not necessary that one (and only one) text-book should be placed in the position of authority in demonstrative

geometry; nor is it necessary that there should be only a single syllabus in control of all examinations. Each large examining body might propound its own syllabus, in the construction of which regard would be paid to the average requirements of the examinees.

Thus an examining body might retain Euclid to the extent of requiring his logical order. But when the retention of that order is enforced, it is undesirable that Euclid's method of treatment should always be adopted; thus the use of hypothetical constructions should be permitted. It is equally undesirable to insist upon Euclid's order in the subject-matter; thus a large part of the contents of Books III. and IV. could be studied before the student comes to the consideration of the greater part of Book II.

In every case, the details of any syllabus should not be made too precise. It is preferable to leave as much freedom as possible, consistently with the range to be covered; for in that way the individuality of the teacher can have its most useful scope. It is the competent teacher, not the examining body, who can best find out what sequence is most suited educationally to the particular class that has to be taught. A suggestion has been made that some Central Board might be instituted to exercise control over the modifications made from time to time in every syllabus issued by an examining body. It is not inconceivable that such a Board might prove useful in helping to avoid the logical chaos occasionally characteristic of the subject known as Geometrical Conics. But there is reason to doubt whether the authority of any such Central Board would be generally recognised.

Opinions differ as to whether arithmetical notions should be introduced into demonstrative geometry, and whether algebraic methods should be used as substitutes for some of the cumbrous formal proofs of propositions such as those in Euclid's Second Book: for opinions differ as to the value of strictly demonstrative geometry, both for training and for knowledge. Those teachers who do not regard algebraic methods as proper substitutes for geometrical methods might still use them, as well as arithmetical notions, for the purpose of illustrating a proposition or explaining its wider significance. It is the general opinion of the Committee that some association of arithmetic and algebra with geometry is desirable in all cases where this may be found possible; the extent to which it may be practised will depend largely upon the individual temperament of the teacher.

Every method of teaching demonstrative geometry has to face the difficulties inevitably associated with any complete and rigorous theory of proportion. In the opinion of the Committee, not merely is Euclid's doctrine of proportion unsuited for inclusion in elementary work, but it belongs to the class of what may be called university subjects. The Committee consider that the notion of proportion to be adopted in a school course should be based upon a combination of algebraical processes with the methods of practical geometry.

As regards examinations in geometry, the Committee consider that substantial changes in much of the present practice are desirable. In most, if not in all, of the branches of mathematics and especially in geometry, the examination ought to be arranged so that no candidate should be allowed to pass unless he gives evidence of some power to deal with questions not included in the text-book adopted. Such questions might comprise riders of the customary type, arithmetical and algebraical illustrations and verifications, and practical examples in accurate drawing and measurement. The Committee consider the latter of particular importance when the range is of an elementary character; some influence will be exercised upon the teaching, and some recognition will be given to the course of practical geometry that should be pursued in the earlier stages.

The Committee are of the opinion that, in the processes and explanations belonging to the early stages of these subjects, constant appeal should be made to concrete illustrations.

In regard to arithmetic, the Committee desire to point out, what has been pointed out so often before, that, if the decimal system of weights and measures were adopted in this country, a vast amount of what is now the subject-matter of teaching and of examination could be omitted as being then useless for any purpose. The economy in time, and the advantage in point of simplification, would be of the greatest importance. But such a change does not seem likely to be adopted at present; and the Committee confine themselves to making certain suggestions affecting the present practice. They desire, however, to urge that teachers and examiners alike should deal with only those tables of weights and measures which are the simplest and of most frequent practical use.

In formal arithmetic, the elaborate manipulation of vulgar fractions should be avoided, both in teaching and in examinations; too many of the questions that appear in examination papers are tests rather of mechanical facility than of clear thinking or of knowledge. The ideas of ratio and proportion should be developed concurrently with the use of vulgar fractions. Decimals should be introduced at an early stage, soon after the notion of fractions has been grasped. Methods of calculation, accurate only to specified significant figures, and, in particular, the practice of contracted methods, should be encouraged. The use of tables of simple functions should be begun as soon as the student is capable of understanding the general nature of the functions tabulated; for example, the use of logarithms in numerical calculation may be begun as soon as the fundamental law of indices is known.

In regard to the early stages of algebra, the modifications (both in teaching and in the examinations) which are deemed desirable by the Committee are of a general character.

At first, the formulae should be built on a purely arithmetical foundation, and their significance would often be exhibited by showing how they include whole classes of arithmetical results. Throughout the early stages, formulae and results should frequently be tested by arithmetical applications. The arithmetical basis of algebra could be illustrated for beginners by the frequent use of graphs; and the practice of graphical processes in such cases can give a significance to algebraical formulae that would not otherwise be obtained easily in early stages of the subject.

In passing to new ideas, only the simplest instances should be used at first, frequent reference still being made to arithmetical illustrations. Advance should be made by means of essential development, avoiding the useless complications of merely formal difficulties which serve no other purpose than that of puzzling candidates in examinations. Many of the artificial combinations of difficulties could be omitted entirely; the discussion of such as may be necessary should be postponed from the earlier stages. Teachers and examiners alike should avoid matters such as curious combinations of brackets; extravagantly complicated algebraic expressions, particularly fractions; resolutions of elaborate expressions into factors; artificially difficult combinations of indices; ingeniously manipulated equations; and the like. They have no intrinsic value or importance; it is only the mutual rivalry between some writers of text-books and some examiners that is responsible for the consideration which has been conceded to such topics.

If general simplification either on these or on similar lines be adopted, particularly if graphical methods are freely used, it will be found possible to introduce, quite naturally and much earlier than is now the case, some of the leading ideas in a few subjects that usually are regarded as more advanced. Thus the foundations of trigonometry can be laid in connection with the practical geometry of the subject-matter of the Sixth Book of Euclid. The general idea of co-ordinate geometry can be made familiar by the use of graphs; and many of the notions underlying the methods of the infinitesimal calculus can similarly be given to comparatively youthful students long before the formal study of the calculus is begun.

[The Appendix consisted of "Two Suggested Schedules of Experimental Geometry," submitted by Mr. Eggar and Professor Perry respectively. They are published in *The School World* for October.

The Report was followed by a paper (read by Mr. Siddons) and a discussion, in which Professors Schuster, Turner, Lodge, Culverwell, Perry, Forsyth, F. Purser, J. Purser, Dr. Macaulay, Messrs. A. W. Eggar, and C. Godfrey took part. The discussion is given, we believe verbatim, in the *Northern Whig* for 17th September. This enterprising paper gave verbatim reports of the papers and discussions throughout the whole meeting—a record enterprise in journalism. W. J. G.]

MATHEMATICAL NOTES.

114. [D. 6. d.] (*a*) With reference to the points raised in paragraph 108 of the current issue of the *Mathematical Gazette*, I offer the following remarks.

A recommendation by the Association as to trigonometrical and hyperbolic notations would justly carry considerable weight, but it is questionable whether the time has yet come for stereotyping any usage.

A good deal could be said against the continental *arc sin* and *arc cos hyp.* In length they are clumsy, in etymology archaic (for the modern definitions of *sin*, etc., as ratios, ignore the arc), and they have the great disadvantage of indicating the inverse of a function in a way not applicable to other inverse functions. The usual British $sin^{-1}x$, $cosh^{-1}x$, etc., have at any rate the merit of being in line with the general $f^{-1}(x)$ for the inverse of $f(x)$. Another general method, which avoids the danger of confusion with the negative index of a power, is that of inverting the spelling: thus nis x is the inverse of sin x. This method has been proposed by Mr. J. W. L. Glaisher (I think) and others. The difficulty as to pronouncing is easily avoided by giving nis x the name of antisin x, etc., which is in line with *antilog* x, already in use here. The extended use of the prefix *anti* (which I learnt from an American pupil) is, I believe, common in America. I have found it very convenient. Antisin is more easily pronounced than Inverse sine.

As to "Complaint 2," I think the practice of professional computers has very little weight as an example for students. There may be an advantage in an artificial process to one who has to use it mechanically thousands of times every year, while to one who only uses it a few score of times in his life the balance of advantage is all the other way. I have no doubt that this is the case as regards Tabular Logarithms. It is not necessary to reprint the tables with negative indices. A very little practice enables the student instinctively to subtract 10 from the tabular log in copying it out, and to add on 10 to his result before looking at the tables. The alternative to this is to go through an awkward bit of reasoning each time the tables are used, or to memorize a complicated rule of thumb, to no good purpose. R. F. Muirhead.

114. [D. 6. d.] (*b*) With reference to the "Two Minor Complaints" of C. S. J. (ii. p. 188, July) permit me to offer the following remarks.

1. The so-called hyperbolic functions should strictly be termed *rectangular-hyperbolic* functions. In my "Algebra of Coplanar Vectors" I proposed and used the term *excircle* for the octosyllabic Rectangular Hyperbola (a curve as it were in a certain sense a circle turned inside out) and for the functions the term *excircular*. In accordance with this the notation for the particular functions should have been *exsin* or *ēsin*, *excos* or *ēcos*, etc., both appropriate and easy to pronounce. I did not venture however to discard the usual notation *sinh*, etc., regarding it as not too inconvenient to make a change imperative, and at the same time too firmly rooted to be easily disestablished. For oral purposes I think the teacher would not find it confusing to use *exsin*, etc., while writing *sinh*, etc.

2. As to Tabular Logarithms, it is perfectly easy (as De Morgan always advised his pupils to do) to knock off the superfluous 10 in extracting the logarithm from the tables and to write down the true logarithm, e.g. 1·7, 0·7, $\bar{1}$·7, $\bar{2}$·7, etc., where the tables give 11·7, 10·7, 9·7, 8·7, etc., respectively. This done, computation is facilitated, and all difficulty in class teaching avoided. The difficulty of printing accurately negative characteristics (if a real one) may be left to be settled between editor and printer.

ROBT. B. HAYWARD.

REVIEWS.

Calcul des Triquaternions: Thèse presentée à la faculté des Sciences de Paris pour obtenir le grade de Docteur ès Sciences Mathématiques, soutenue April 1902. By M. GASTON COMBEBIAC, Capitaine du Génie. Paris: Gauthier-Villars.

M. Combebiac has developed without undue prolixity in an elaborate and lengthy thesis of 122 pages the principles of a new system of analysis. The complexity of the method may be inferred from the length of the memoir, but it is not easy in a brief abstract to convey an adequate impression of the author's ingenuity or to explain the multifarious methods of the calculus.

A triquaternion is an expression of the form

$$r = q + \omega q_1 + \mu q_2,$$

in which q, q_1, and q_2 are quaternions, while ω and μ are extraordinaries commutative with quaternions and satisfying the relations

$$\omega^2 = 0, \quad \omega\mu = -\mu\omega = \omega, \quad \mu^2 = 1.$$

It may be easily shown that the triquaternions are associative in multiplication and that all algebraic functions of triquaternions are themselves triquaternions. Three selective symbols, G, P and L, analogous to the symbols S and V of quaternions, play an important part in this theory. They are defined by the equations

$$Gr = Sq\,; \quad Pr = \omega Sq_1 + \mu Vq_2\,; \quad Lr = Vq + \omega Vq_1 + \mu Sq_2\,;$$

and obviously

$$V = Gr + Pr + Lr.$$

M. Combebiac employs particular cases of triquaternions as symbols of points, planes, and lines. Thus the triquaternion $\omega\alpha + \mu x$ is the symbol of a point – the extremity of the vector $x^{-1}\alpha$ drawn from an assumed origin, x being a scalar and α a vector. In like manner $\mu\alpha + \omega x$ represents the plane whose quaternion equation is $S\rho\alpha = x$. Again $\alpha + \omega\beta$ is the symbol of a right line when the condition $S\alpha\beta = 0$ is satisfied; in fact $\alpha = t(\rho_2 - \rho_1)$ and $\beta = tV\rho_1\rho_2$, t being a scalar, if the vectors ρ_1 and ρ_2 drawn from an assumed origin terminate upon the line. More generally when α and β are not at right angles, M. Combebiac regards $\alpha + \omega\beta$ as the symbol of the linear complex whose quaternion equation is $S\rho_1\rho_2\alpha + S(\rho_2 - \rho_1)\beta = 0$, ρ_1 and ρ_2 being variable.

Hence it appears that Pr is the symbol of a plane, and Lr is the sum of the symbols of a complex $Vq + \omega(Vq_1 - \gamma)$, and of a point $\mu Sq_2 + \omega\gamma$, where γ is an arbitrary vector. By imposing the condition $SVq(Vq_1 - \gamma) = 0$, Lr is expressible as the sum of the symbols of a line and of a point, and, further, the vector γ may be uniquely selected so that the point lies upon the line. Lr is called the *élément linéaire* of the triquaternion. Thus a triquaternion is reducible to the sum of a number and of the symbols of a point, a line, and a plane.

Similarly, a quaternion is the sum of a number and a vector; but it is also the ratio of a pair of vectors; it may be employed as a point-symbol or a plane-symbol, and as an operator it turns vectors in its own plane through a determinate angle and alters their lengths in a given ratio, or, more generally, it produces a linear transformation of points or planes. To interpret a quaternion as a point, it is only necessary to observe that the extremity of the vector Vq/Sq drawn from an assumed origin is determinate when the quaternion q is given.* Further, we may suppose this point loaded with a weight Sq, and thus

* *Trans. R.I.A.* xxxii. Part I.: "The Interpretation of a Quaternion as a Point Symbol."

all the elements of a quaternion are utilized when it is taken to represent a weighted point. With this convention the equation of a plane may be written $Sqp=0$, q being a variable and p a given quaternion; and because this plane is determinate when p is given, we may regard a quaternion as the symbol of a plane as well as of a point. Thus quaternions are as well adapted as triquaternions for dealing with projective properties of points and planes. Indeed, quite apart from the greater simplicity of quaternions, the advantage seems to lie with them, as the principle of duality is rather obscured by the two different symbols in triquaternions for points and planes. This is very evident from the work on p. 76 of the memoir, for new notations have to be introduced in order to extend to points certain fundamental relations connecting planes.

Every element of a quaternion is essential to the idea of a weighted point or to Hamilton's beautiful conception of the ratio of two vectors, but it seems to be difficult to get a triquaternion to work up to the limit of its twelve elements. The calculus rather branches off into three distinct sections according as we are concerned with points, lines, or planes. For lines, the section is Clifford's bi-quaternions. Let us compare the results of multiplying a point by a quaternion and a triquaternion. In quaternions taking a point $\omega+\rho$ and multiplying it by a given quaternion q we obtain the new point $q\ (\omega+\rho)$ of weight $\omega Sq+Sq\rho$ and situated at the extremity of the vector $(\omega Vq+Vq\rho)/(\omega Sq+Sq\rho)$. This is a particular homographic transformation, the general transformation being effected by a linear quaternion function operating on $\omega+\rho$.* Similarly multiplying a point $\mu\omega+\omega\rho$ by a triquaternion $q+\omega q_1+\mu q_2$ the result is a new triquaternion, $\omega q_2+\omega(\omega q_1+q\rho-q_2\rho)+\mu\omega q$, and not a point unless $q_2=0$, $Vq=0$, and $Sq_1=0$. When these conditions are satisfied, the triquaternion reduces to $Sq+\omega Vq_1$, and the effect of operating on a system of points is to translate them as a whole. As another example, if we apply to a point m the operator $r(\quad)r^{-1}$ in which r is an arbitrary triquaternion, we obtain (p. 35) an expression of the form $m'+\omega y$, where m' is a point and y a numerical quantity. M. Combebiac remarks (p. 35), "nous ne voyons pas de signification géometrique simple à ces transformations." If, however, the condition is imposed that the operator converts points into points, the triquaternion reduces to the form $q(1+t\mu)+\omega q_1$, where t is a scalar and where $SqKq_1=0$. In this case the operator gives a body a general displacement and a uniform dilatation. This is rather disappointing. We might have expected more from a triquaternion with its twelve parameters.

Little need be said about the section of the calculus which deals with lines and linear complexes, as the method is similar to that of bi-quaternions, or *octonions*, as they have been called by McAulay. We submit, however, that it is hardly sufficient for the development of bi-quaternions to regard them as simply a compound of quaternions and of a commutative ω whose square is zero, for we find (Octonions, p. 55) that $\phi(\omega a)$ is not equal to $\omega\phi a$ where ϕ is the general function corresponding to an ordinary linear vector function. In quaternions the general linear function fq is a sum of products Σaqb, but this is not so in octonions or in triquaternions, and a new principle has to be introduced. In octonions $\phi(a+\omega\beta)=\phi_1 a+\phi_2\beta+\omega(\phi_3 a+\phi_4\beta)$, where ϕ_1, ϕ_2, ϕ_3, and ϕ_4 are ordinary linear vector functions, and consequently †

$$\phi\omega a=\phi_2 a+\omega\phi_4 a \text{ while } \omega\phi a=\omega\phi_1 a.$$

On account of this additional complexity, I think it is extremely doubtful if bi-quaternions have any advantage over quaternions, even in the treatment of screws and complexes. It is, for example, quite as easy to deal with the symbol (μ, λ) employed in the appendix to the new edition of Hamilton's Elements (vol. ii., p. 390) as with the equivalent symbol $\lambda+\omega\mu$, and occasionally we require functions homogeneous in two or more μ's (as in the case of the sexiant), which do not seem to be directly obtainable from bi-quaternion forms. It is worthy of notice that Hamilton was quite familiar with the conception of a localized vector, Clifford's *rotor*. I am not aware that he published anything on the subject, but a short manuscript on *tractors* may be found in the library of Trinity College, Dublin. Hamilton's tractor is identical with Clifford's rotor, the name being connected with the idea of a force along a definite line of action.

* For $\omega=0$ we obtain the transformation of points at infinity. These remain at infinity if $Sq\rho=0$, and thus we fall back on the case of multiplying a quaternion into a vector in its plane.

† M. Combebiac does not discuss this question.

In an appendix to his memoir, M. Combebiac has collected many elegant examples and interpretations of the products of triquaternions of special types. The paper will doubtless be read by many with great interest, for the invention of geometrical algebras seems of recent years to have become quite a popular amusement. We may mention those of Gibbs, Heaviside, Macfarlane, McAulay, Hyde, and Major Ronald Ross of malaria fame, but in some ways the calculus of triquaternions is the most ambitious of all. C. J. JOLY.

Theory of Differential Equations. Part III. **Ordinary Linear Equations.** A. R. FORSYTH. 1902. Pp. xvi. + 534.

This, the fourth and latest volume of a complete work on Differential Equations, contains the theory of ordinary linear equations treated from the point of view of the theory of functions. In consequence of this limitation several branches of the subject have been omitted, such as the formal theory, the theory of invariants and covariants, and the application of the theory of groups. The volume opens with a general discussion of linear equations with uniform coefficients, and the existence of integrals which are regular functions in the domain of an ordinary point is established. These integrals are called 'synectic,' as the term 'regular' is reserved for integrals of the form $(z-a)^r[\phi_0+\phi_1 \log. (z-a)+\ldots+\phi_k\{\log. (z-a)\}^k]$ in the vicinity of a single point $z=a$. Chapter II. is concerned with fundamental systems of integrals near a singularity and the corresponding fundamental equation; groups of integrals associated with a multiple root are considered in relation to the elementary divisors. Chapter III. deals with regular integrals in the neighbourhood of a particular singularity and the indicial equation is introduced. The two succeeding chapters contain a discussion of Fuchsian equations, having all their integrals regular in the vicinity of every singularity, and equations possessing algebraic integrals. Chapter VI. determines limits to the number of regular integrals, and contains an account of Lagrange's adjoint equation. In Chapter VII. we come to essential singularities and integrals of the types called 'normal' and 'sub-normal,' together with Poincaré's development of Laplace's definite-integral solution. Chapter VIII. contains an account of infinite determinants which will be new to most readers, and their properties are developed so far as they are required for the solution of linear equations. The last two chapters deal with equations possessing coefficients of particular types, simply-periodic, doubly-periodic, and algebraic; in connection with the latter, automorphic functions are introduced. On the whole the results are mostly descriptive and of a widely general character. Instances of special functions occur in dealing with the hypergeometric equation and with Lamé's equation.

Mathematical and Physical Papers. By Sir GEORGE GABRIEL STOKES, Bart., M.A., etc. Vol. III. Pp. viii., 416. (Cambridge University Press.)

Everyone will be pleased that the publication of Sir George Stokes's scientific papers has at last been resumed. The papers contained in this volume were originally published in the years 1850-52; the three longest are "On the Effect of the Internal Friction of Thirds on the Motion of Pendulums," "On the Colours of Thick Plates," and "On the Change of Refrangibility of Light." To the last a short addition has been made, explaining how the author's view of the nature of fluorescence was subsequently modified; with the exception of this, and a few occasional footnotes, the papers have been reprinted as they originally appeared.

Linear Groups: with an Exposition of the Galois Field Theory. By L. E. DICKSON, Ph.D. Pp. x., 312. (Leipzig: B. G. Teubner.)

The author of this book writes with complete mastery of his subject, to which, indeed, he has made many original contributions. The theory of groups, *per se*, is one of the most abstract fields of mathematical speculation; and there are not many, even among mathematicians, who take real pleasure in it for its own sake. But its influence upon analysis is so far-reaching that some acquaintance with it is becoming almost indispensable; even in problems of pure geometry, questions of group-theory force themselves upon the attention. For this and other reasons it is, perhaps, to be regretted that Professor Dickson has preferred to expound his theme in the most abstract possible way. Thus the Galois field is reduced to a system of 'marks' devoid even of arithmetical significance; and even the

polyhedral groups are treated without any reference to the systems of rotations from which they take their names. Moreover, the style of the work is extremely concise; so that, although it is self-contained, and does not imply any previous knowledge of the subject on the part of the reader, it demands strenuous attention, and may prove discouraging to the beginner.

But if the reader perseveres he will find plenty to reward him. The first part, while mainly introductory, contains a very interesting account of quantics irreducible to an integral modulus, with a table of known results in this theory: the second and principal part deals with groups regarded as sets of permutations of 'marks' defined by linear substitutions. The substitutions may be integral or fractional in form, they may be homogeneous in the marks, or otherwise: this gives one broad scheme of classification. Sub-divisions are established by means of various analytical properties of the groups, or perhaps it would be more proper to say of the algebraic forms in which they are presented. For instance, we have the Abelian linear group, with a generalisation of it; the linear homogeneous group with a quadratic invariant, and so on. To what extent this classification is intrinsic it is not quite easy to say: the method of 'marks' is of course extremely general, but after all it is a particular form of representation, and its results are to a certain extent qualified by the mechanism which has produced them. They may, of course, lead immediately to theorems which belong to pure group-theory: for instance, we may be able to prove that a certain group, defined by substitutions, is simple. This is undoubtedly an intrinsic quality of a group; and the fact that so many of the known simple groups of composite order can be represented by linear substitution-groups shows the power and generality of the method.

Hard to read as Professor Dickson's work undoubtedly is, it has the cardinal merit of being the work of an original investigator, and the subject is accordingly presented as it is viewed by one individual mind. At the same time the author is familiar with what has been done by others, so that his readers can easily turn to contemporary research in this field. It is remarkable to see how often reference is made to the work of Camille Jordan: the permanent value of his great treatise on substitutions is thus once more duly emphasised.

It is impossible, in a brief review, to give any detailed account of Professor Dickson's treatise; it must suffice to recommend it to all students of group-theory as an extremely able presentation of that aspect of the subject with which it deals.

G. B. MATHEWS.

Slide Rule Notes. By LT.-COLONEL H. C. DUNLOP, R.F.A., and C. S. JACKSON, M.A. Pp. 65. 1901. (Simpkin, Marshall & Co.)

This is the approved text-book on the Slide-Rule at the Royal Military Academy (Woolwich). Its authors are evidently masters of the art of slide-rule calculation, and show in this text-book their power of seizing on every available artifice in pushing the resources of the instrument to the utmost.

The most useful feature of the book seems to us to be its excellent collection of miscellaneous examples. Here samples are given of problems in the many branches of engineering and physics, which, needing only an accuracy of ·2% to ·5%, are solved on the slide rule with far more expedition and with less liability to accidental error than when log tables are used. We quote (in an abbreviated form) one or two "examples" typical of the wide range of selection:—"4. Obtain the % composition of $KClO_3$." "13. Refraction increases the altitude of a star by $T=57\cot a$ seconds nearly. Find y for 10°, 20°, etc." "35. Probable error of a shot being P, the chance that an error $<nP$ is $q=\sqrt{1-\cdot 75n^2}$ approximately, work out results for $n=\cdot 1$, ·25, etc." "43. Deflection of beam is $D=\frac{5}{32}\frac{wl^3}{Ebd^3}$ calculate D for an oak beam with given dimensions." "49. When an electric current liberates x c.c. of (H and O) mixed gases $C=\frac{xn273}{\cdot 1733\times 760(273+T)}$, calculate C from the given data." Since each of the examples as given at present refers to a distinct problem, it would be an advantage in the use of the book as a student's exercise book if the number of examples were increased three or four-fold by a repetition of examples on the same problem.

The chapters dealing with the theory of the instrument are doubtless attuned

to the mathematical taste of the Woolwich student, and are throughout "correct," yet to our mind they savour too much of bookiness, and show some tendency to manufacture "bookwork." For instance, a formal proof is given of the fact that when using a slide rule the % error due to limitation in power of accurate graduating and reading the scale is the same whatever part of the rule has been employed. Now any intelligent student can realize this truth when, with the rule before him, it is pointed out that the intervals 1 to 1·02, 1·02 to 1·04 on the scale are identically equal to the intervals 5 to 5·1, 5·1 to 5·2, and practically equal to the intervals 9 to 9·2, or 9·6 to 9·8, and so on.

The oppression produced by this tendency to formality is increased by the uninviting character of the figures which, owing to lack of detail and of any attempt at proportion, are unreal and unconvincing.

On the other hand, the book refers to many interesting and useful points.

Thus the methods given for solving a quadratic $x^2+px+q=0$ from the fact that $\alpha\beta=q$ and $\alpha+\beta=-p$ are interesting, and moreover yield very accurate results. In the simplest case the cursor is set to q on the A scale, the slide is then adjusted until the reading its left end marks on A added to the reading on B below the cursor gives p numerically; these readings are the numerical values of the roots.

Analogous methods are given for solving cubic equations.

Allusion is made to the log-log scale introduced by Roget in 1815, which we understand is now being brought forward again in a practical form by Professor Perry. The use of this scale in combination with the log-scales of the ordinary rule obviously gives at sight the value of a number raised to any desired power; for the rule, in mechanically adding log-log x to log n, gives log (n log x), *i.e.* log-log x^n.

On the whole this text-book is much to be commended. It is thoroughly modern in method, and of course free from the use of "gauge points" and other paraphernalia of the old-fashioned slide-rule treatises. F. R. BARRELL.

Gleichen's Lehrbuch der geometrischen Optik. (Teubner, Leipzig.) 1902.

This work aims at bringing the teaching of Optics into touch with the practical problems of the subject. There are, in the earlier part, faults of arrangement, and the author has sacrificed something to the desire for completeness, but, on the whole, the book is well adapted to its purpose.

The treatment of Aberrations of the first order, in which considerable use is made of Abbé's work, prepares the way for the subsequent discussion of the properties of Optical instruments and the preparation of formulae for exact computations. The practical bearing of the subjects discussed is never lost sight of, and, for this reason, the chapters on Orthoscopy and Curvature of the Image are worthy of special attention. But the interest and value of the work centre in the chapters dealing with the telescope, microscope, and the photographic objective, and it is evident that no trouble has been spared to make them complete.

The author gives the various conditions that have been propounded for telescopic objectives, and in many cases gives formulae or tables for their calculation as well as the details of a number of objectives.

The chapter on microscopes is not so complete, owing to the refusal of information as to the details of modern constructions, but the author has been able to secure details of most modern photographic objectives—the figures being taken from actual lenses, and in some cases the results of calculating their errors.

The work is one which is worthy of the attention of all who are interested in Applied Optics. S. D. CHALMERS.

The Tutorial Arithmetic. By W. P. WORKMAN, M.A., B.Sc. 3/6. (The University Tutorial Series.)

Arithmetic. By R. HARGREAVES, M.A. 4/6. (Clarendon Press.)

Arithmetic. By A. VEITCH LOTHIAN, M.A., B.Sc. (Blackwood and Sons.)

A South African Arithmetic. 3/6. By HENRY HILL, B.A. (Methuen and Co.)

The latest to come of this batch of new Arithmetics was Mr. Workman's *Tutorial Arithmetic*, for which the rest have been kept waiting. It was well worth waiting for and we consider it distinctly the best. Those who have made the acquaintance of the Kingswood School "Arithmetic Prize Papers," either in the small book by the same author (J. Hughes & Co., 1895) or from time to time in separate sheets, will expect a very thorough treatment of the Theory of Arithmetic, especially of the parts of it connected with Scales of Notation, and will be prepared for a certain amount of arithmetical revelry in the long-drawn-out sweets of recurring decimals. They must be very hard to please if they are disappointed with the fare provided for them. But the author's well-known tastes have not prevented his giving a thorough all-round treatment, which renders his book one to be heartily recommended to the notice of all mathematical students, under which denomination it is hoped that many mathematical teachers will not disdain to consider themselves included. A few words from the preface will give a good idea of the aim and scope of the book and of the terse vigour with which it is written: "Unusual prominence is given to questions of Theory. Formal completeness is beyond the reach of an English author whose soul is vexed perforce with Weights and Measures and the barbarous mechanism by which they are manipulated, who is condemned also by the tyranny of an examination system to waste the time at his disposal in elaboration of the details of Complex Fractions, abstract and concrete, and other subjects of similar inelegance and uselessness. Moreover, further to militate against completeness, there exists an extraordinary prejudice against the use of literal symbols, which are supposed to be the inviolable property of Algebra." The claim that Theory is given unusual consideration is well supported by the fundamental treatment of the simple processes, the discussion of the Commutative and Distributive Laws, the treatment of Evolution and the chapter devoted to Congruences. The last will be one to which most readers will turn. It will be found a very interesting and useful introduction to the Theory of Numbers, particularly to students who wish to become expert in the solution of the Arithmetic Prize Papers. But while prominence is thus given to Theory it must not be supposed that practical work, including under that head clear and rapid methods and good arrangement, is neglected. Approximation is adequately treated, and side remarks on points of historic or other interest are added as occasion arises for them. Those who attended the meeting of the Association last January may be interested in the fact that the author follows Dr. Mackay (*Theory and Practice of Arithmetic*, Chambers) in reproducing Catalan's treatment of $\sqrt{3}$ as a limit. There is so little to be said which is not praise that it seems scarcely worth while to point out a few items of treatment in which we do not agree with the author. We do not like the arrangement of the work on the top of p. 163. Possibly it is due to the difficulty of explaining in print as contrasted with oral teaching. We always insist that the fraction shall appear as a whole in each successive step, thus:—

$$\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{4+\frac{1}{5}}}}=\cfrac{1}{2+\cfrac{1}{3+\frac{5}{21}}}=\cfrac{1}{2+\frac{21}{68}}=\frac{68}{157}.$$

If the process is allowed to be done *in bits*, it should be only after pointing out that the generating fraction can be obtained by a process somewhat like that of forming convergents, except that the quotients are written in reverse order: thus

writing	5	4	3	2
we obtain by an easy process		21	68	157

and as the connexion of the formation of a Continued Fraction with the H.C.F. process is explained on the same page, it might with advantage be pointed out, as is done by Henslow in his *Scholar's Arithmetic*, that when H.C.F. methods are used for reducing a fraction to its lowest terms, the final division of numerator and denominator by the H.C.F. is unnecessary; for the value of the fraction in its lowest terms can be found by means of the quotients. To take an example from Henslow, the reduction of $\frac{1615}{1159}$ to its lowest terms.

By the G.C.M. process (which he arranges conveniently in a single column, as here on the right) he finds the successive quotients 1, 2, 1, 1, 5, 2.

1615	1
1159	2
456	1
247	1
209	5
38	2
19	

Using these in reverse order with a subsidiary (1),

$$2 \quad 5 \quad 1 \quad 1 \quad 2 \quad 1,$$

he obtains a set of numbers he call numerals, thus

$$2, \quad 11, \quad 13, \quad 24, \quad 61, \quad 85,$$

the value of the required fraction being $\frac{85}{61}$.

We differ slightly from the author in his estimate of the value of the so-called "Italian" method of division, which we value as a precious thing and insist on with vigour throughout all the classes under our charge. When it is taught from the first by teachers who thoroughly understand and appreciate it, we believe it to conduce more to accuracy than the common method, which we hope is following to extinction the old scratch method which it supplanted.

Mr. Hargreaves' is also an excellent work. Here, too, the 'shop' or 'complementary' method for subtraction, the master-key to rapid computation, is well explained and applied to "Italian" division. Theory receives due attention. Approximation is adequately treated: we notice with interest that, like Mr. Workman, the author gives a section to "Approximation by Aliquot parts," a method which has not yet received the attention it deserves. It is to be regretted that this excellent work is disfigured by the retention of an antiquated method for the extraction of cube root.

Opinions differ as to whether cube root is worth the attention usually given to it, but there can hardly be two opinions by those thoroughly versed in Horner's method as to its superiority to all others, both for rapidity of execution and easy generalisation, for the solution of cubic and higher equations. Of Mr. Lothian's book much the same may be said. It seems well and carefully done and is up-to-date as regards 'shop' subtraction, and approximation. It is beautifully printed; everything that differences of type and good arrangement of work can do seems to have been done. But here again we have to regret that Horner's method has not replaced the ancient one given.

There is not much novel or striking in the fourth book on our list except its name, and the introduction of weights and measures peculiar to the Cape. When we first saw that "100 lbs. = 1 Cape hundredweight" we began to hope that enlightened colonists had taken practical steps towards the establishment of a decimal system; but our hopes were dashed by finding further on that 1 morgen $= 2\frac{1}{9}$ acres; 3 bushels = 4 schepels = 1 muid or sack. So that there seem to be school children even more to be pitied than the victims of our barbarous system.

Logarithms. By F. Glanville Taylor, M.A., B.Sc. (Longmans, Green & Co.)

An excellent practical treatise, with just enough theory given to render the principles on which the practice is founded intelligible to a beginner. The author justifies his claim to knowing most of the difficulties and pitfalls that students are likely to encounter, and has taken much trouble in dealing effectively with them. The type used is excellent, a return being made to the pre-Huttonian characters with heads and tails, as in the reprints of Lalande and of Barlow's Tables edited by De Morgan. A small folding card, *Table of Five Figure Logarithms* (Oliver & Boyd, 6d.), has the same clear type.

Another excellent Arithmetic comes into our hands almost simultaneously with the proof-sheets of the above. **An Arithmetic for Schools.** By J. P. Kirkman, M.A., and A. E. Field, M.A. 3s. 6d. We have consequently not time to go through it carefully, but we see with pleasure an appendix of 12 pages devoted to *squared paper*, the diagrams of which are nicely executed. This is an interesting and useful innovation. In multiplication and division the authors still cling too much to traditional methods of work. But we know too well the excuses they can plead. In all five of the above works on Arithmetic we have to notice one omission which is doubtless due to the nature of examination papers.

We could well spare much Arithmetical Gymnastic for a simple chapter on the nature of logarithms and their application to numerical calculation. Sufficient theory

to make the application intelligible can be given by means of square root by interpolating *fractional* in a series of *integral* powers of any base—and for preference in starting. *Smith and Beman* in their Higher Arithmetic (Ginn & Co.) gave a serviceable section (pp. 109-121) on logarithms, and we should like to see their example followed by English writers.

The Story of Euclid. By W. B. FRANKLAND, M.A. 1s. (George Newnes.) We heartily recommend this book to the attention of mathematical teachers. The author seems to have accomplished a somewhat difficult task in a very satisfactory manner, and to have succeeded in avoiding the two pitfalls that beset any such attempt—on the one hand the too free use of technical terms which would have repelled all but mathematicians from reading it; on the other, the insipid vagueness which would be caused by the entire avoidance of them. A preliminary sketch of what is known of the achievements of Thales, Pythagoras, and other predecessors of Euclid who made his work possible, is followed by an account of the Alexandrian school, and an estimate of the famous "Elements." But the author does not stop here—he also gives the story of Euclid's successors down to Proclus, 'the last of the Greeks,' from whom he makes a long quotation. We then have an interesting account of the earliest printed Euclids, and a sketch of the rise of Modern Geometry. Finally we come to Lobachewski and Riemann of whom portraits are given. *An excellent shilling's worth.*

Geometric Exercises in Paper Folding. By T. SUNDARA ROW. Edited and revised by BEMAN & SMITH. 4s. 6d. (Kegan Paul, Trench, Trübner & Co.) The original Madras edition was noticed in No. 3 of *The Mathematical Gazette* (Dec. 1894). Profs. Beman and Smith having had their attention drawn to it by a remark of Klein's,* and having been convinced of its usefulness to teachers and students, obtained permission to bring out an American edition. This is embellished by upwards of 20 beautifully executed photographic reproductions of paper folds. In our opinion the author pushes paper-folding far beyond the stage at which it is really helpful in school teaching, but perhaps an enthusiast for any particular way of looking at a thing does good service in riding his hobby hard and thus attracting attention to its powers. He gives other odds and ends of mathematical lore likely to be helpful to teachers and through them stimulating to students. His geometrical illustration of the identity

$$(\Sigma r)^2=\Sigma r^3$$

might be made still more geometrical, for there exists a fairly obvious geometrical proof that $1+2+1=2^2$; $1+2+3+2+1=3^2$, and hence that

$$2+4+2=2^3, \quad 3+6+9+6+3=3^3$$

and so on. E. M. LANGLEY.

The Elements of Geometrical Drawing. By HENRY J. SPOONER, C.E. (Longmans, Green & Co., 1901.) 3s. 6d. Pp. xxxix. 298.

Mr. Spooner's book is entitled *Geometrical Drawing*, but the author seems to intend it as a book of reference on various subjects. He informs us that 3 barleycorns = 1 inch, and that a stone is 14 pounds, but is 8 pounds in the London Meat Market, and he quotes the value of π to 32 significant figures.

In his attempt to be complete, the writer touches on many points far better left alone, and his numerous definitions, which show a surprising lack of sequence, can only serve to worry the reader.

There are many errors, some of which are accidental slips, but one finds it difficult to account for the following property of the Ellipse—"The product of the focal distances of any point P on the curve is equal to the square of half the major axis." This property the author fixes in the minds of his readers by means of a special figure drawn for the purpose.

Some of the printed formulae supplied for reference are wrong, and others are quoted in a form most unlikely to appeal to the intelligence of the student.

The author mentions in his preface that the footnotes need not trouble the

* "Hermann Wiener has shown how by paper-folding we may obtain the network of the regular polyhedra. Singularly, about the same time a Hindu mathematician, Sundara Row, published a little book . . . in which the same idea is considerably developed. The author shews how by paper-folding we may construct by points such curves as the ellipse, cissoid, etc." Klein's *Famous Problems of Elementary Geometry.* (Ginn & Co.)

beginner. It is to be feared that some of them are likely to trouble a careful reader who looks to them for explanation. On page 126, for instance, the foot-note reads, "The student will thoroughly understand this expedient when he knows that the sum of the focal distances of any point on the curve is always equal to the major axis." Yet "this expedient" for drawing an ellipse makes no use of the foci.

On page 154 the equation $x=\frac{a}{b}\sqrt{y^2+b^2}-a$ is given as the equation of the hyperbola referred to its centre.

The figures are in most instances carefully and neatly drawn, and often show at a glance the method of construction. The book certainly contains some neat geometrical constructions, and especially some elegant approximate methods, but the writer apparently does not know when his constructions are accurate and when only approximate. This is conspicuous on page 27, where an accurate construction for a regular pentagon is followed by the remark, "With good drawing the above method gives a result near enough for many practical purposes." Lower down the page an approximate construction is given without remark.

Plane Geometrical Drawing. By R. C. Fawdry, M.A. (E. & F. N. Spon, Ltd., 1901.) 6s. net. Pp. xi. 185.

This book is intended primarily for army candidates, and exhibits the subject as affording an opportunity for the student to make practical use of his knowledge of Pure Geometry. Many of the explanations are clear and concise, the arrangement is good, the examples numerous and well chosen, and there is an absence of that large amount of repetition frequently found in such text-books.

Word-references to Euclid might with advantage be substituted for many of the *number-references*, which in some cases are scarcely intelligible. For instance (page 11) the hint for proving the construction adopted for bisecting the angle between two lines, without producing them to their point of intersection, reads thus—"*Proof*—By Euc. I. xxix." We may here remark that some of these references are incorrect.

On page 16, it would appear that π is merely a short way of writing 22/7. One notices with regret such expressions as "whose side is $\sqrt{8''}$," "of length $\sqrt{2''}$." The following method of denoting the equality of the areas of two rectangles is open to criticism:

$$\text{“}FE,\ FD=FA,\ FB.\text{”}$$

This quotation, by the way, is taken from a piece of deductive geometry which, as it stands, is quite unintelligible. "The circle we require," should read, "The circle through A, B, G." Mr. Fawdry concludes with 50 pages of army examination papers, omitting the questions on Solid Geometry. We welcome the book as an honest attempt to place the subject of *Geometrical Drawing* in its true relationship to *Pure Geometry*.

Practical Solid Geometry. By Capt. E. H. de V. Atkinson, R.E. (E. & F. N. Spon, Ltd., 1901.) 7s. 6d. Pp. x. 124, and 21 Plates.

In this second edition of the Woolwich Text-book, revised by Major B. R. Ward, R.E., except for the addition of a few problems, very little alteration appears to have been made, and several obvious mistakes remain uncorrected. It is unfortunate that the publishers have not seen their way to distribute the figures throughout the book.

After his introductory chapter, the author proceeds immediately to the simplest cases of the projections of solids. The next three chapters are devoted to a consideration of the more important problems connected with the intersections of lines and planes, using the "Index System," which shows the *elevations* of points by suffixes in the *plan*. Then the author deals with the more difficult cases of projections of solids, and finally gives a short chapter on Isometric Projection.

To one who has already grasped the elementary principles of Solid Pure Geometry, this book will prove extremely useful and interesting, but the arrangement is frequently confusing and the explanations not always clear. What are we to understand by "Solids are assumed to be *transparent*, and the invisible

edges are represented by dotted lines"? Here is another startling sentence,—"Note that the distance between two parallel planes is the vertical distance between the elevations of the two planes." It seems strange to find in Chapter III., after the fact has been continually assumed, an attempt to show that the projection of a straight line is a straight line, based upon Euclid's definition of a straight line. Problems XV. and XVI. of Chapter V. should be entirely re-written and figures 57 and 58 re-drawn; then, perhaps, there will be no need for the remarks about the difficulty to a beginner. The former problem is the determination of the angle between two given planes M and N. The two pairs of contours (0, 0) and (10, 10) determine two points r_0 and p_{10} on the line of section. No other point on this line is required, and through p_{10} a plane O should be drawn at right angles to the line r_0p_{10}; then the angle required is the angle between the lines (O, M) and (O, N). In the printed figures there are two contours (10, 10) which should be marked (9, 9). These are unnecessary and misleading, and they cannot be drawn until the figure is completed. There is a curious mistake in figure 70, the worked-out solution for drawing a regular octahedron with two intersecting faces inclined at given angles to the *H.P.* The dotted lines in the finished diagram show that the author is quite unconscious of the fact that he has constructed his solid *underneath* the two given planes. In fact, the dotted lines really represent those edges which are nearest to the eye of an observer looking down upon the *H.P.* Of the two examples given of Isometric Projection, that of a regular octahedron is not calculated to exhibit the *utility* of the method.

There is, however, thoroughly sound and practical work in the book, and it seems a pity that Major Ward has not taken advantage of his opportunity of improving it.

First Stage Building Construction. By BRYSSON CUNNINGHAM, B.E. (University Tutorial Press, 1901.) 2s. Pp. viii. 240.

A capital little text-book on Building Construction for the use of beginners. It is not a mathematical book, but the author has, in a few instances, given a simple explanation of the theory on which the construction is based, thus introducing the reader to the nature of the stresses in the different members of a structure, without considering the magnitudes of such stresses. The book is clearly written and well illustrated.

Exercises in Graphic Statics. By G. F. Charnock, Assoc. M.Inst.C.E. (J. Halden & Co., 1900.) Part II. 52 sheets of diagrams with printed explanations.

Part II. of this work is devoted to the graphical treatment of stresses in beams, cantilevers, and girders, and, as in Part I., the examples are taken from actual practice. Much of the subject matter is, of course, of a technical character, but it includes a great deal that would be interesting to the student of mathematics. This is not a book to be placed in the hands of a beginner, for he is not likely to acquire a knowledge of sound principles from these pages, where the explanations are frequently obscure and bewildering. Take, for instance, the author's exposition on sheet 2 of various methods for determining the moment of resistance at a vertical section of a loaded horizontal beam of breadth b inches and depth d inches. Denoting the stress per square inch at the extreme fibre of the section by S, he indicates how the moment of resistance M for the whole section may be determined by adding together the moments of resistance for the various horizontal layers. He abandons this method without making the summation, and suggests that the process may be simplified by "taking the mean or resultant stress per square inch on the various layers, which of course $=\frac{S}{2}$, and assuming this to act uniformly over the whole area." He says that this resultant will be found by a graphical construction to pass at a distance $z=\frac{2}{3}\times\frac{d}{2}$ from the neutral axis. "Then," we read, "moment of resistance in compression = area above neutral axis × mean stress × mean arm z, and this result × 2 for corresponding moment in tension below neutral axis gives the moment of resistance M for the whole section, or

$$\left(\frac{b\times d}{2}\right)\times\left(\frac{S}{2}\right)\times\left(\frac{2}{3}\times\frac{d}{2}\right)\times 2=\frac{Sbd^2}{6}."$$

Surely it would be too much to expect a beginner to understand from this what is really intended and why the process gives the correct result.

Lower down the same page we have the method of determining the moment of resistance from the resistance area of the section.

As the author has not developed further the direct method of determining the moment of resistance by adding together the moments obtained from the individual layers, the reader is not introduced to the consideration of the moment of inertia of a section until sheet 6. There Mr. Charnock determines an approximate value for the moment of inertia I. of a section of the rectangular beam, and assuming that the true value is $\frac{bd^3}{12}$, he notices that $I=\mu y_0$, when μS is the moment of resistance already obtained and y_0 the distance of the extreme fibre from the neutral axis. This is considered sufficient to convince the student that the formula $I=\mu y_0$ is true for any beam, whatever the shape of the section.

One would forgive Mr. Charnock for not proving the important principles on which he bases his methods, if he only presented them clearly and in some order, so as to rivet his pupils' attention upon them; but one cannot pass over such proofs as these. Now it is quite easy to demonstrate in a general way that the total normal stress on either side of the neutral axis may be expressed in each of the following equivalent forms:

(i) $\frac{S}{y_0}\times$ moment of area of section on that side;

(ii) $S\times$ resistance area on that side.

Also, as the normal stress above the neutral axis must form a couple with the normal stress below, we have the equivalent results:

(i) The neutral axis contains the centre of gravity of the section;

(ii) The resistance area above the neutral axis is equal to that below $=A$ (say).

Again, it is also easy to show that the moment of resistance of the section can be expressed in each of the following forms:

(i) $\frac{S}{y_0}\times I$;

(ii) $S\times A\times a$;

where a is the distance between the centres of gravity of the two areas A.

The product $A\times a$ may usually be readily determined with sufficient accuracy from the diagram, as Mr. Charnock says, and I may be deduced by multiplying by y_0.

In the case of the rectangular beam,

$$A=\frac{1}{4}bd \text{ and } a=\frac{2}{3}d, \text{ so that } M=\frac{Sbd^2}{6} \text{ and } I=\frac{bd^3}{12}.$$

The book is useful for the sake of its practical examples of bending moment, shearing force, and other diagrams, but we would recommend the student to acquire a mastery of the principles involved by a study of other works.

W. J. Dobbs.

Primer of Geometry. By H. W. Croome Smith, B.A. 2s. 1902. (Macmillan.)

This little book seems to be intended for the teacher and not for the pupil. In these days, when there is some prospect of being freed from the fetters of Euclid, such a book should be welcomed by those who are considering the future of geometrical teaching. Not the least valuable part of the book is the admirable preface. There we find an eloquent plea for the abandonment of Euclid, and for the separation of the course of theorems from that of constructions. The definitions and the notes thereupon are carefully done. One may not agree with all the details, but the general idea is very suggestive. It is somewhat surprising, in so revolutionary a work, to find Euclid's definition of a tangent preferred to the 'limit' definition. The word *duplicates*, to describe figures equal in all respects, is a good one, but is hardly likely to replace the term *congruent*.

A very natural order of propositions is given in which the theorems are made independent of the methods of construction, and is very like the orders adopted in the best continental and American geometries. Several questions will occur to the thoughtful reader, and their answers are still open to doubt.—If pupils are

to go through a course of accurate drawing and measurement, will it not be more convenient to take the proportions on areas (Euc. I. 34-48) before those on the circle?—If we use hypothetical constructions, we ought strictly to say, *e.g.*, "Let AD be the straight line through A at right angles to BC" instead of using the imperative:—"From A draw AD at right angles to BC." Surely the imperative is in most cases the more convenient form, and it may be used provided the difficulty is pointed out.

There are no riders or numerical examples.

Euclid, Book. XI. By R. LACHLAN, Sc.D. 1s. (Edward Arnold.)

At the request of several teachers Dr. Lachlan has added this volume to the other books he brought out some time ago. The result can hardly be said to be satisfactory. The figures are badly drawn, there is a sad lack of perspective, and shading might have been used with advantage in some cases. Propositions 6 and 8 are proved in the old way. The proof in which there is symmetry about the plane containing the two parallel lines is to be preferred—it is less artificial than the old proof.

A. W. SIDDONS.

Differential Calculus for Beginners. By ALFRED LODGE. Pp. xxv–278. Price 4s. 6d. 1902. (George Bell & Sons.)

This very attractive little book is intended to provide an easy introduction to the Calculus for those students who have to use it in their practical work, and to make them familiar with its ideas and methods within a limited range. Designed primarily to meet the wants of engineering students it would prove a very stimulating text-book if placed in the hands of an intelligent pupil who has a good working knowledge of elementary Algebra and Trigonometry and some slight acquaintance with the methods of Co-ordinate Geometry. Thirty years ago the approach to the Differential Calculus lay through Todhunter's work, and the malignant hideousness of that first chapter will to many be an indelible memory. But to-day the path of the student is bestrewed with flowers, —the crooked has been made straight and the rough places plain.

Sir Oliver Lodge contributes an introduction to his brother's book. This introduction may be looked upon in the light of an alluring prospectus on which the advantages of an intellectual investment in the Calculus are set forth without being at all overstated. "For in so far as things remain constant they are stagnant: every kind of activity involves variable quantities, and therefore to every kind of activity the Differential Calculus is applicable." Instances are given from familiar notions of speed, slope, expansibility, vibrations, propagation of waves, which all require for their proper expression the ideas and notation of the Calculus.

As would naturally be expected, the plotting of graphs receives a great deal of attention. Excellent diagrams are inserted both in the text and among the answers, so that complete familiarity with this method of geometrical representation cannot fail to be secured. From the very beginning, when the curve $y=x^2$ is plotted, it is expressly stated that there is no need for the abscissa and ordinate to be set off on the same scale; but that different convenient scales should be selected having regard to the numerical range of the co-ordinates. This of course implies that the student has had some preliminary experience in plotting. Surely at first it should be strongly urged that an equation between x and y involves one, and one only, graph of a definite shape (though its size may vary), and the gradient defined as the tangent of the angle which the direction of the curve at any variable point makes with the axis of x is correctly depicted without distortion in that graph. Later on this conception may very well be extended, and a relation between x and y may be supposed to give rise to an infinite number of graphs by varying the ratio of the scales. [A circle may in this way become an ellipse of any shape.] This difference of scales, however, leads to difficulty when plotting the circle of curvature (pp. 117-9).

The proof of Taylor's theorem (pp. 173-4) is very satisfactory and as simple and well arranged as it is possible to make it. There is a slight misprint on p. 174, line 20, for "zero when $\theta=0$," one should read "zero when $z=0$."

Maxima and minima are introduced fairly early, leading to easy questions of the Parcel-Post-Regulations type. The statement that a position of symmetry will always make the function a max. or min. will call to mind a very thorough

discussion of this principle at a meeting of the A.I.G.T. over a paper by the Rev. J. J. Milne.

The differentiation of a^x is very ingeniously done on pp. 54, 55. This is always a difficulty with beginners, and in order that no time should be lost it is best to proceed thus:—

$$\text{If } y=(1+x)^{1/x},\ \log y=\{\log(1+x)\}/x=\left(1-\frac{x}{2}+\dots\right) \text{ if } x<1$$

$$=1, \text{ if } x=0.$$

Therefore Lt. $(x=0)y=e$.

There are several alternative methods given for the resolution into partial fractions. The forcing method of division involves only the simplest algebra, and might almost be taught in the nursery.

A very useful discussion is given of compound-interest growth ($Dy \propto x$), which is of frequent occurrence and great importance. One can well imagine the delirious enjoyment with which a senior wrangler would evaluate such a formula as $T=100e^{\cdot 046t}$ (p. 67). Most probably he would mentally consign the tea to a place where the temperature needs no formula!

The graphic solution of equations is interesting and well done. It would have been better to have given an example of a cubic in the text where no analytical work is possible, as students will not fly to graphic solutions when the treatment of a quadratic by algebra is so easy and familiar. The proof from Cremona (pp. 91-4), to which the author drew attention at the M.A. meeting in January last (attributed by Mr. E. M. Langley to Thomas Carlyle), deserves to be well known and studied.

Simple integrals (just so far as they are the inverse of differentials) are given throughout the book, the notation for which is $D^{-1}y$, $D^{-2}y$, $D^{-3}y$ in contradistinction to Dy, D^2y, D^3y.

In determining the form of a curve near the origin a graphic method is given on pp. 204-5, which decides what terms of the equation between x and y may and may not be neglected. This seems very ingenious and effective.

The proof for $\rho=rdr/dp$ on p. 254 involves certain assumptions connected with the substitution of the circle of curvature for the curve itself which often require a great deal of mental digestion. Possibly to some minds the following method is more satisfactory:

Differentiate with respect to s the relation $p=r\sin(\phi-\theta)$, and we have

$$\dot{p}=\dot{r}\sin(\phi-\theta)+r\cos(\phi-\theta)(\dot{\phi}-\dot{\theta}),$$

in which the first and third terms pleasurably cancel out as $\dot{r}=\cos(\phi-\theta)$ and $r\dot{\theta}=\sin(\phi-\theta)$. Hence, etc.

In conclusion, if we are to try the new doctrine what more admirable instrument to our hand is wanted than such an introduction to the Calculus as this? Let the examination syllabus be overhauled, and revised, and all but the elements of Analytical and Geometrical conics be ruthlessly cut out: so shall knowledge of the Calculus prevail! A new generation will arise that knows not Joseph Wolstenholme. But it will have what are after all the real tests of sound training, far greater keenness and facility in manipulation and far higher appreciation of useful practical work.

R. F. DAVIS.

Urkunden zur Geschichte der Mathematik im Mittelalter und der Renaissance. Edited by MAXIMILIAN CURTZE. Pp. x, 336. 16 m. 1902. (Teubner.)

This volume forms the twelfth of the series of original texts published under the direction of the greatest of mathematical historians—Moritz Cantor. To the master, on the occasion of the golden jubilee of his doctorate, it is dedicated by his grateful friend and pupil, Herr Maximilian Curtze. It contains the Latin text of the *Liber Embadorum* of one Abraham bar Chijja Savasorda, translated into Latin by Plato of Tivoli, the oldest known translator of works from the Arabic. In England it is pretty certain that one could hardly persuade a publisher even to discuss the idea of such a translation. In Germany, a great house like that of Teubner undertakes it, and the blandishments of the German Foreign Office are

at the disposal of the editor to enable him to borrow two precious manuscripts from the National Library at Paris.

The brief biography of Plato of Tivoli, quoted from Wüstenfeld, tells us that he was "born at Tivoli, lived in Spain, there learned Hebrew and Arabic, and translated from both languages works on mathematics and astronomy, *but very badly.*" We hasten to add that the last assertion the editor nails to the counter as a base calumny. One would also gather that the task of translation was no more financially successful in those days in Spain than it is in the present century in Great Britain. Plato omits the introduction and the epilogue to Savasorda's work. The missing portions would seem to reproach the Jews for their ignorance of the rules of geometry, and for their inexcusable blunders in elementary calculation. Evidently our friend Plato had personal experience of the children of Israel, and was so far in their hands that the offending passages were excised. The task of Savasorda was to teach his erring brethren the elements of mensuration. The following is a type of the question he sets his readers: *Given the area of a square* (*or rectangle*) *minus* (*or plus*) *the sum of its sides, find the sides and area.* This is worth knowing, for it involves the solution of a quadratic. Now the algebra of Muhamed ben Musa Alkarismi, translated by Gerard of Cremona, is supposed to have introduced the western world to that mystery. But Savasorda lived at the end of the eleventh and the beginning of the twelfth centuries, which dislodges Gerard from his lofty pre-eminence (Rouse Ball, p. 171). If you would know the diameter of a circle, this is how you set about it: *Cumque circuli embadum sciveris et ejus diametri longitudinem nosse volueris, tres partes de* 11 *embado superaddas, et diametri multiplicationem invenies, cuius summae radix diametri longitudinem continebit.* At the request of the publishers the Latin appears on one side of the page with the German on the other. This arrangement, as the editor slyly observes, will be found useful to many.

The second part of the volume deals with the correspondence of Regiomontanus, Giovanni Bianchini, Jacob von Speier, and Christian Roder. It forms quite an E.T. reprint. Their artless confidences have reference to the secrets and mysteries of spherical geometry, astronomy, etc. Some of Jacob's answers read very quaintly: *Velatas aviculas Pannum, quo cenam nostram ornari voluisti, mensuravi, et proportione bona* 64 *brachia cum reperi, cuius pretium* 35 *ducatorum et unius recte duxi.* Regiomontanus is occasionally didactic, as when he proves that "it is not necessary" that a quadrilateral be inscriptible in a circle. He will then rattle off at a sitting some sixteen conundrums to his faithful friends. Then he ends his letter: *Sed quo ruit calamus! Nimium forsitan fatigaberis, vir optime, elegendo tantas litteras.* Truly, those were the days of "the grand style"! We must heartily thank Herr Curtze for a most entertaining volume, and look forward to the second part with great expectations.

General Investigations of Curved Surfaces of 1827 and 1825. By K. F. Gauss. Translated with notes and a Bibliography by J. C. Morehead and A. M. Hiltebeitel. Pp. viii, 127. $1\frac{3}{4}$ dollars. 1902. (The Princeton University Library.)

We owe a debt of gratitude to the Princeton University authorities for providing the funds necessary for the production of this translation. This handsome volume is a worthy tribute to what Darboux has called one of the chief titles of Gauss to fame, the *Disquisitiones generales circa superficies curvas* of 1827, which is still the most finished and useful introduction to the study of infinitesimal geometry. It is a matter of regret that financial considerations have so far prevented our great publishing houses from embarking upon such a venture as the issue of a series of translations of the great mathematical classics. The Germans have realised to the full the stimulus to the student and the teacher afforded by the study of such a paper as the above as compared with that derived from the ordinary text-book summary. In physical science good work has been done in the republication of original papers by the Alembic Society. The American Book Company has even improved on the idea by issuing volumes containing papers on a subject in their historical order; *e.g.* "The Wave Theory of Light," Huygens, Young, & Fresnel; "The Second Law of Thermodynamics," Carnot, Clausius, & Thomson. In mathematics the Open Court Company (Chicago) has published De Morgan's *Elementary Illustrations of the Differential and Integral Calculus*, and his *Study and Difficulties of Mathematics*; Lagrange's

lectures on *Elementary Mathematics*; Dedekind's papers on *Continuity and Irrational Numbers*, and on the *Nature and Meaning of Numbers*, and so on. It is not improbable that a translation of Gauss's *Disquisitiones Arithmeticae* will be one of the next books to be added to this list. It is a disgrace to the English-speaking mathematical world (or the subtlest of compliments to their Latinity) that no English translation of this masterpiece exists. The most complete series of classical memoirs that has yet appeared is Ostwald's *Klassiker der exacten Wissenschaften*. The paper of Gauss under notice was translated into German by Wangerin, and forms the fifth volume of this really excellent series of mathematical, physical, and chemical memoirs; each volume contains between 50 to 100 pages, and costs but a few pence. They are as cheap as the Open Court volumes are dear.

Messrs. Morehead and Hiltebeiter have translated the abstract of the *Disquisitiones* presented to the Royal Society of Göttingen in 1827, and the paper of 1825 on which the abstract was based. The papers of 1825 and 1827 show, as they point out, the manner in which the ideas developed in the mind of the author. Both papers contain the fundamental properties of what is called Gauss's measure of curvature. Geodesic coordinates in 1825 are replaced by the general coordinates p, q in 1827, thus introducing a new method, and employing Monge's principles. This paper was published by Monge in his *Application de l'Analyse à la Géometrie* (1850). The translators supply twenty-eight pages of notes on the 1827 paper, and the bibliography contains a list of 343 papers dating from "the thirties" to 1901.

Lehrbuch der Combinatorik. By Dr. Eugen Netto (Pp. 260). 1901. (Teubner.)

One of the latest additions to the excellent series of monographs published in the *Sammlung von Lehrbucher auf dem Gebiete der mathematischen Wissenschaften* is from the pen of Professor Netto of the University of Giessen, whose name is familiar in connection with the substitution theory. It is a common complaint among students that in questions on what Prebendary Whitworth so aptly called "Choice and Chance," that they never know exactly where they are. Although much of the glorious uncertainty which lingers in their minds after attempting the solution of a problem may be almost entirely due to careless or ambiguous wording, one cannot help feeling that if the student is left in the air with regard to these and kindred problems it is because of the cursory treatment which this branch of mathematics receives. The interest of the subject is practically inexhaustible. Such a volume as this draws on arithmetic, algebra, analysis, and probabilities for its material; the questions which are set are some of great historical interest; some are as amusing as they are exasperating, and almost all of them require a clear brain and a judicial temperament. For instance, the number of ways of writing the words in the line

Tot tibi sunt dotes, Virgo, quot sidera coelo,

without disobeying the laws of metre (caesura excepted), were given at various dates as 1022, 2196, 3276, 2580, 3096, and 3312. Which is correct we leave our readers to determine. Among the more familiar of the problems treated by Professor Netto is that of the fifteen school-girls, and that of the eight queens none of which are *en prise*. The pretty solution given in *Nature*, p. 427 (1899), has evidently escaped the author. A series of questions is given leading up to the theory of the partition of numbers, and an account follows of Sylvester's determination of the coefficient of x in the development according to ascending powers of x of the expression $(1-x^a)^{-1}(1-x^b)^{-1}(1-x^c)^{-1}\ldots$.

Two chapters are devoted to "combinations of the third class," which the French call *ternes*, and which were first suggested by Steiner, who applied them in his theory of the double tangents of quartics.* If n objects are arranged three by three, so that any two objects appear in one triplet and only one, such an arrangement is called a system of ternes. For example, of the numbers 1, 2, 3, 4, 5, 6, 7, the triplets in question are 123, 145, 167, 246, 257, 347, 356; there remain 28 "dreier" of the system, which are called "freie," free ternes or triplets. But when we proceed to the quarternes or "vierer," which are in the above case,

* *Journ. f. Math.* 1853, p. 181.

1247, 1256, 1346, 1357, 2335, 2367, 4567, there are no free quarternes. The ternes of a system of numbers of the form $6N+1$ or $6N+3$ have been given by Reiss* and Moore.† A careful collection of formulae brings to a close a valuable monograph, which will be found of great interest to the teacher and the student alike.

Grundlinien der Politischen Arithmetik. By DR. MORITZ KITT. Theil I. Zinseszins- und Rentenrechnung. Pp. 78. 1901. (Teubner.)

This little volume will be found of interest to teachers in higher schools of commerce, being written for the *Handelsakademien* and *Hohereren Handelslehranstalten*, which take such a prominent part in the German educational system. The contents are closely limited to the programmes of these schools, and treat in a somewhat condensed manner what we may call, for want of a better name, the "elementary algebra of finance." Compound interest, annuities, stocks, insurance, etc., are briefly treated from the theoretical point of view, with applications and exercises for solution. Tables are given of the logarithms of $\left(1 \pm \frac{\text{rate}}{100}\right)$ from $\frac{1}{4}$ by quarters to ten per cent.; of the powers from two to fifty of the same numbers; we also have four different tables of mortality and the fundamental tables for life annuities and insurances at $3\frac{1}{2}$ to 4 per cent. This excellent manual will be found well adapted for the purpose for which it is designed.

Culegere de Probleme. Compiled by I. IONESCU, etc. Pp. 520. 1901. (Gobl Bucuresti.)

This is an excellent collection of problems in Arithmetic, Algebra, Trigonometry (Plane and Spherical), with their solutions, arranged for the purposes of the "real" and "special" lycées of Roumania. They are as we should say "up to Scholarship standard," and are interesting so far as they bear witness to the thorough nature of Mathematical teaching in the schools between the Danube and the Pruth. With a little practice the reader will find that he is quite able to read Roumanian Mathematical problems and solutions. M. Ionescu is well known as an engineer, and is the able editor of our name-sake the *Gazeta Mathematica*; his colleagues in the compilation under notice are MM. Ţiteica, Ioachimescu and Cristescu, whose names have travelled as far West as the columns of the *Educational Times*. The solutions are sometimes rather sketchy, but are quite sufficient for the purposes of the average student.

Algebraische Gleichungen. By E. BARDEY. Pp. 420. 1902. (Teubner.)

This handsomely bound and printed volume covers no more than the usual space given to equations in such books as the Algebras of Chrystal, Hall & Knight, or Charles Smith. Beginning with equations with one unknown—quadratics, and equations reducible to quadratics, it concludes with equations of the second degree of three and four unknowns. Certain types are taken, and solved at full length. Then follow numerous examples of the types, carefully arranged in order of difficulty, hints and references being given where deemed advisable. The teacher may find the collection useful, but the claims on the time of the student in this country are too exacting for the detailed perusal of this volume to be possible, even if it were necessary. Life is short, and a book like this is terribly long—in proportion to the ground it covers. On the other hand, the book is from cover to cover detailed, thorough, and complete. But to write it was to break a butterfly on a wheel.

Problems in Electricity. By R. WEBER; translated from the third French edition by E. A. O'KEEFFE. Pp. 351. 1902. (Spon.)

The compiler of this collection of problems on all branches of electrical science, some 700 in number, and ranging from the elements of the subject to higher practical questions on series, shunt, and compound dynamos, motors, glow, and arc lamps, lighting installations, telegraphy and telephony, has gathered together a capital collection which should be of the greatest use to the student. Solutions are given throughout. Although the scope of the questions is largely outside our range, yet a cursory glance at even the first ten pages has shown some six or seven errors in the results. The questions are well arranged

* *J. f. Math.* 1859, p. 326. † *Math. Ann.* 1893, p. 271.

and should be found extremely useful, but we fancy that there is room for considerable revision of the solutions.

Der Naturwissenschaftliche Unterricht in England inbesondere in Physik und Chemie. By Dr. Karl T. Fischer. Pp. viii and 94. 3 m. 60. 1901. (Teubner.)

This elegantly got-up volume hardly comes within the scope of the *Gazette*, being an account, impartial and with but few exceptions correct, of the provisions made for the teaching of science—physics in particular—in English schools. Dr. Fischer, who is a Privatdozent and teacher of Physics in the Royal Technical High School at Munich, was sent by the Bavarian Government to make a study of the position of science in the curricula of English schools. A good deal of attention is paid to the Heuristic method of teaching, with which Professor Armstrong's name is indissolubly linked. The general conclusion is not, however, of the most favourable character. The most striking quotation made by the author is from some remarks of Mr. Earl, to whom the science side of Tonbridge School is so deeply indebted. He prefers boys from the classical side to boys from the modern; "they prove better, being of higher standard in character."

Elementary Geometry. By W. C. Fletcher. Pp. iv., 84. 1s. 6d. 1902. (Arnold.)

The revolt against formality is already beginning to bring forth fruit in the shape of books such as this before us, embodying most of the recommendations of the various committees that have been dealing with the question of reform in our mathematical teaching. Within the compass of 80 pages or so, the author gives us a guide and a summary to the whole of the substance of Euclid I.-IV. and VI., with the exception of the "elegant but unimportant proposition IV. 10." It contains "the irreducible minimum of geometrical knowledge, less than the whole of which is not worth considering as knowledge at all." He confesses that the book is "frankly unorthodox—there is no reference to axioms or rather postulates . . . this is not work for a boy, but for mature and subtle minds." With the sets of riders are incorporated suggestions for drawing, plotting, and measurement, in accordance with the principle that the early stages of abstract geometry must be as far as possible associated with concrete illustration. We cannot speak too highly of Mr. Fletcher's work. It is of course a skeleton which must be provided with flesh by teacher and pupil in judicious combination. It has been constructed with admirable skill, and every teacher will find it most suggestive.

Geometrical Deductions. Books I. and II. By James Blaikie and W. Thomson. New Edition. 2s. 6d. 1900. (Simpkin, Marshall.)

We are glad to see that this handy course of "training in the art of solving riders" has reached a second edition. Among those to whom the authors acknowledge valuable assistance and suggestions are Messrs. R. F. Davis, Tucker, Hayward, Mackay, and Milne, whose names are as familiar to us in the columns of the *Gazette* as that of Mr. Blaikie himself.

Die Elemente der Analytischen Geometrie, zum Gebrauche an höheren Lehranstalten sowie zum selbststudium. Vol. II. **Die Analytische Geometrie des Raumes.** By Dr. F. Rudio. Third Edition. Pp. 186. 1901. (Teubner.)

This excellent little introduction to the elements of geometry of three dimensions, which has reached its third edition in ten years, is written by the Professor of Mathematics at the Zurich Confederal Polytechnicum. The simplest cases alone are treated, the general equation of three variables being excluded. It contains some five hundred examples of a simple nature, and unlike most foreign publications of the kind, it can boast of a good index, in addition to the detailed summary of the chapters. It is clearly the work of an able and experienced teacher, and appears to be well adapted to the needs of the budding engineer or physicist.

Katechismus der Differential- und Integralrechnung. By F. Bendt. 2nd Edition. Pp. 267. 3m. 1901. (Weber, Leipzig.)

This is an exceedingly easy introduction to the elements of the calculus, and is well worth perusal by the teacher who is anxious to take a class through a first

reading of the subject. Even the simpler forms of differential equations are not neglected.

Katechismus der Algebraischen Analysis. By F. BENDT. Pp. 154. 2m. 50. 1901. (Weber, Leipzig.)

This handy little summary covers the same ground as the larger Hall and Knight, condensed and arranged on much the same lines as the above volume of the same series by the same author. Both volumes are prettily bound in an "Originalleinen-band."

Solutions of the Problems and Theorems in Smith and Bryant's Geometry. By C. SMITH. Pp. 230. 1902. (Macmillan.)

This key to the edition of Euclid recently published by the authors will be found useful to the private student. We have, however, detected more than one instance in which the solutions do not correspond to the exercises, for instance, in the cases of Nos. 16, p. 127; 1, 2, 4, pp. 140-141; 22, p. 146; 27, p. 147; 155, 156, p. 190. Perhaps Mr. Smith will issue a slip containing the necessary corrections, as there possibly may be a few more which have eluded a cursory inspection. For the solutions themselves we have nothing but praise.

The Beginnings of Trigonometry. By A. C. JONES. Pp. vi., 144. 2s. 1902. (Longmans.)

This little book is remarkable for the early prominence given to trigonometrical and logarithmic tables, so that "the student gets from the start a knowledge of tables needful in the science laboratory." Special stress is laid on the novel ideas to which the student is introduced, such as the revolution of a line and the corresponding increase of angle—ideas which must prove of great use to him in his early lessons in physics. For this purpose the book is well planned and arranged, a signal instance of careful arrangement being the author's treatment of the ambiguous case.

Repertorium der höheren Mathematik (Definitionen, Formeln, Theoreme, Literatur). Part II. Geometrie. By E. PASCAL. Pp. x., 712. 12m. 1902. (Teubner.)

The advanced student and the teacher will welcome this translation into German (by Ober Lieutenant Schepp) of the second volume of the Synopsis of higher mathematics published two years ago by Professor Pascal of Pavia. Here he will find, without the labour of turning up a number of journals or text-books, a definition, a formula, a theorem, and a perfect treasure-house of bibliographical information. A summary will indicate the scope of the Repertorium: Geometry of continuous and discontinuous figures; conics; quadrics; general theory of plane algebraical curves; connexes in the plane; cubic plane; planes of the fourth order; general theory of algebraical surfaces and of algebraical curves in space; curves on quadrics; gauche curves; surfaces of various orders; ruled surfaces; Plückerian geometry; enumerative geometry; infinitesimal properties of curves and surfaces; geometry of the particular curves; analysis situs (topology); polyhedra, Riemann surfaces; projective geometry in space of more than R_n dimensions; infinitesimal and intrinsic geometry of an R_n, linear or of constant curvature; absolute and non-Euclidean geometry. A collection dealing with such a variety of matter may perhaps be excused for having in the first imprint so many errors as to require eight pages of supplementary corrections and additions. We have noticed one slip in the text which has escaped the notice of the author. He says (p. 625) that the Riemannian geometry "gilt weder das Postulat V noch Postulat VI." But the sixth postulate is what we call Axiom 12, which holds good in Riemann's system, and in the single elliptic of Clifford and Klein it is only the final clause that is rejected. There is an excellent index of authors and of subjects in addition to the full table of contents.

ERRATA.

Page 188, line 23—*for* $\sqrt{a^2-x^2}$ *read* $\sqrt{x^2+a^2}$.

" 189, " 10—*for* or better *read* whether.

" 189, " 27—*for* $4\sinh 3\theta$ *read* $4\sinh^3\theta$.

BOOKS, ETC., RECEIVED.

Theorie der Algebraischen Funktionen einer Variabeln und ihre Anwendung auf algebraische Kurven und Abelsche Integrale. By Drs. HENSEL and LANDSBERG. pp. xvi, 708. 26 marks. 1902. (Teubner, Leipzig.)

Spezielle Algebraische und Transcendente Ebene Kurven. By GINO LORIA. Translated into German by Fritz Schütte. Zweite Hälfte. pp. xxi, 417-744. 10 marks. 1902. (Teubner.)

Die Elemente der Analytischen Geometrie. Part II. Die Analytische Geometrie des Raumes. By Dr. F. RUDIO. 3rd edition. pp. x, 186. 1901. (Teubner.)

Applied Mechanics for Beginners. By J. DUNCAN. pp. xii, 324. 2s. 6d. 1902. (Macmillan.)

Cours d'Algèbre. By B. NIEWENGLOWSKI. 2 vols. pp. 386, 510. 4th edition. (Colin.)

An Elementary Treatise on Theoretical Mechanics. By ALEX. ZIWET. I. Kinematics, pp. viii, 182. II. Statics, pp. viii, 184. III. Kinetics, pp. viii, 236. In one volume. 21s. net. 1901. (Macmillan.)

Elementary Geometry. By J. ELLIOTT. pp. xii, 268. 4s. 1902. (Swan Sonnenschein.)

Sur Une Classe de Surfaces (D. ès Sc. Thesis). By M. L'ABBÉ M. DE MONTCHEUIL. pp. 75. 1902. (Gauthier-Villars.)

Higher Mathematics for Students of Chemistry and Physics. By J. W. MELLOR. pp. xiv, 544. 12s. 6d. net. 1902. (Longmans, Green.)

Aritmetica Generale e Algebra Elementaire. By G. PEANO. pp. 144. 1902. (Paravia, Torino.)

Proceedings of the Edinburgh Mathematical Society. Vol. XX. Session 1901-1902.

Notes on the Theorems of Menelaus and Ceva; Constructions connected with Euclid VI. 3 *and* A *and the circle of Apollonius; Geometry of the Isosceles Trapezium and the Contraparallelogram with applications to the Geometry of the Ellipse.* By R. F. MUIRHEAD. pp. 12. (Reprinted from above Proceedings.)

Étude Géométrique des Lignes et des Surfaces en un Point Ordinaire. By P. VANDEUREN. pp. 40. (Lebègue, Bruxelles.)

An Elementary Treatise on Kinematics and Dynamics. By J. G. MACGREGOR. 2nd edition. pp. xvi, 525. 10s. 6d. 1902. (Macmillan.)

The Science of Mechanics. By E. MACH. Translated by T. J. M'Cormack. Second revised and enlarged edition. pp. xx, 605. 9s. 6d. 1902. (Open Court, Kegan Paul.)

The R.M.A. Magazine. July, 1902. (The Mathematical Note Books of a Cadet in 1775.) By C. S. JACKSON.

Review of Barbarin's "La Géometrie non-Euclidienne." By G. B. HALSTED. (*Science*, June 20, 1902.)

The Metric System. By T. C. MENDENHALL. (*Popular Science Monthly*, Oct. 1896.) Issued by the New Decimal Association.

Periodico di Matematica. Edited by G. LAZZERI. Anno XVIII. Fasc. 1, 2.

Gazeta Matematica. Edited by ION IONESCU. July, Aug., Sept., 1902.

Matriculations Algebra. By R. DEAKIN. pp. viii, 455. 2nd edition. 3s. 6d. 1902. (Clive.)

Katechismus der Mechanik. By PH. HUBER. Edited by W. Lange. pp. xiv, 269. 3 m. 50. 1902. (Weber, Leipzig.)

Essai sur l'étude des fonctions données par leur developpement, etc. Taylor. Essai sur les propriétés des fonctions entières. By J. HADAMARD. pp. 215. 1902. (Hermann, Paris.)

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